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Inflation from warped space

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Abstract: A long period of inflation can be triggered when the inflaton is held up on the top of a steep potential by the infrared end of a warped space. We first study the field theory description of such a model. We then embed it in the flux stabilized string compactification. Some special effects in the throat reheating process by relativistic branes are discussed. We put all these ingredients into a multi-throat brane inflationary scenario. The resulting cosmic string tension and a multi-throat slow-roll model are also discussed.

Keywords: D-branes, Cosmology of Theories beyond the SM.

Contents

1. Introduction

Inflation [1 – 3] provides a natural mechanism for creating the homogeneity and flatness of our observable universe. It also gives an elegant way of generating the perturbations [4 – 10] which seed the structure formation. In order for the inflation to last sufficiently long and then successfully exit to reheat the universe, the inflaton has to be held up on a potential for a sufficiently long time. Such a mechanism is achieved most commonly by a potential which is very flat on the top. The required flatness is summarized by the slow-roll conditions. A central problem in inflation has been to find a natural realization of such a flat potential in a fundamental theory. Many years of research in supergravity and string theory indicates that, while such flat potentials may arise in many occasions, they are not generic. Therefore it is important to ask if inflation can happen given a steep potential, while generating a scale invariant spectrum. In this paper we study such a model by making use of the warped space.

Recently warped space has shown its importance in both field and string theory. It has been proposed as one of the few possible explanations to the hierarchy problem [11]. In string theory, such space arise as a consequence of flux compactification [12, 13], and play important roles in stabilizing the extra dimensions and constructing the dS space and inflationary models [14 – 32].

In this paper, we study an interesting use of the warped space in the context of the inflation. Since the infrared (IR) end of a warped space generally has a small warp factor, to a bulk observer at the ultraviolet (UV) end of this warped space, anything trapped in the IR side moves very slowly in the extra dimension. This is because the speed of light traveling in the extra dimension is small in the IR end. In particular this applies to a D3-brane. To a four-dimensional observer, the extra dimension is the internal space and the position of the D3-brane in the extra dimension is a scalar field. Therefore this provides a new mechanism for the scalar field to move very slowly [17, 24, 28].

In terms of the gravitationally coupled scalar field theory, we will be interested in a scalar field with a relativistic kinetic term and rolling down from the top of a steep potential in a warped internal space. The causality restricts the scalar to roll slowly and we will show that it is quite robust against the steepness of the potential. The appearance of the resulting inflation is kind of similar to the slow-roll inflation: the inflaton stays at the top of the potential for a long time before it falls down through fast rolling. However the more detailed nature of these two scenarios are different: for example, in slow-roll inflation, the potential is flat and the inflaton is non-relativistic, while here the potential is steep and the inflaton is ultra-relativistic during inflation. This mechanism is especially interesting in situations where the warped space become necessary and generic, but the flat potentials are not. We will call this type of inflationary models as DBI inflation.

It is important to realize effective field theories of any inflation model in a unified fundamental theory such as string theory. In this paper we are interested in the idea of the brane inflation [33 − 38] in the flux stabilized string compactification [13, 14, 16]. The setup is the orientifold compactification of type-IIB string theory. Besides stabilizing the complex and dilaton moduli, the NS-NS and R-R fluxes also induce warped space (throats) around various conifold singularities. Such warped space carry D3-charges. They attract anti-D3-branes in the bulk and then annihilate them. Because the D3-charge of a throat is discrete in multiples of some integers, D3-branes will generally be created at the IR end of the throat after this annihilation [39, 22].

We will be particularly interested in those throats with large flux numbers. The flux-antibrane annihilation process in such a throat proceeds through quantum tunneling [39, 22]. In the four-dimensional spacetime point of view, the annihilation creates a bubble in the false vacuum. Generally the interior of the bubble is still in a false vacuum, because there may be a moduli potential for the resulting D3-branes, or anti-D3-branes in other places of the compact manifold waiting to be annihilated. If such a moduli potential is flat enough, the slow-roll inflation can happen within the bubble. Under a steep repulsive moduli potential, normally these D3-branes will quickly roll out and our universe cannot live in such a bubble. However since here we have a situation where the D3-branes are trapped in an IR warped space, such a rolling is subject to a causality constraint, namely the DBI inflation can happen.

A multi-throat brane inflation model [28] will be studied in more details. In this model, branes generated as above roll out of the brane (B) throat, triggering the DBI inflation. They reheat and settle down in the Standard-Model (S) throat. We show that such a model can generate the right density perturbations with a direct reheating and a Randall-

Sundrum (RS) warp factor. Subtleties of the relativistic brane reheating [40], and its effect on the density perturbations and on the large flux number are studied. Other possible cases, for example adding an antibrane (A) throat, and a multi-throat slow-roll model are also discussed.

The multi-throat configuration provides a unique opportunity to observe signals of string theory. It gives a hierarchical range of scales. For low string scale such as the RS setup, we may have chance to observe strings in colliders. For throats with high string scale, brane inflation can create cosmic strings [41 − 43]. They may give observable signals in addition to the density perturbations and the spectral index [44 − 51]. We will discuss the corresponding string tension in our cases. We also discuss another way strings are produced during the dS epoch, which are more general but with less tension.

This paper is organized as follows. In section 2, we describe the effective field theory of the DBI inflation. This includes the zero-mode inflation and the density perturbations. In section 3, we embed the field theory in the setup of the flux compactification, where the inflaton dynamics is described by the DBI action of D3-branes in warped extra dimensions. Various constraints coming from the validity of the DBI action and an interesting stringy suppression mechanism on density perturbations are discussed. In section 4, we turn to the reheating process. We emphasize two important processes that arise quite often for the throat reheating in a multi-throat setup – the relativistic brane collision and cosmological rescaling. We put all these ingredients in a multi-throat model in section 5. In appendix A, we discuss branes rolling into a throat, which is used in section 4, and briefly review another DBI inflation model. Appendix B studies how the DBI inflation and slow-roll inflation are jointed in case of a flat potential. This leads to a multi-throat slow-roll inflation model. Cosmic strings produced in different cases are discussed accordingly in section 5 and appendix B.

2. Field theory of DBI inflation

Although the DBI inflation scenario is motivated by string theory, the field theory description of the main process during the inflationary period can be extracted out independently, and is interesting in its own right. This describes a scalar rolling down from a steep potential in a warped internal space. In this section, we will study how inflation arises in this setup and the resulting density perturbations. When appropriate, we will mention its connections to the string model that will be discussed later. A similar type of model was studied in ref. [17, 24], where an important difference is to start the inflaton from the UV side. The resulting inflationary scenario has some qualitative differences and will be compared in the end of appendix A.

For later convenience, we will denote the scalar field as r, which is related to the usual scalar field ϕ through $r(t, \mathbf{x}) \equiv \phi(t, \mathbf{x})/\sqrt{T_3}$ (the constant T_3 is the brane tension). The scalar moves in an internal warped space with a characteristic length scale R

$$ds^2 = h^2(r) g_{\mu\nu} dx^\mu dx^\nu + h^{-2}(r) dr^2 , \qquad h(r) = \frac{r}{R} . \tag{2.1}$$

Here $ds_4^2 = g_{\mu\nu}dx^\mu dx^\nu$ is the four-dimensional space-time metric. It is highly warped near $r \sim 0$. The scalar field $r(t, \mathbf{x})$ can be thought of as a 4-d hypersurface embedded in 5-d space (2.1).

The action which governs the gravitationally coupled scalar is given by

$$S = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}R - T_3\int d^4x\sqrt{-g}\left[h^4\sqrt{1 + h^{-4}g^{\mu\nu}\partial_\mu r\partial_\nu r} - h^4 + V(r)\right]. \quad (2.2)$$

Note that in this paper V has been made dimensionless by pulling out a factor of T_3. The kinetic term in (2.2) may be understood as a generalization of the kinetic term for a homogeneous scalar in flat four-dimensional space-time

$$-T_3\int d^4x\left[h^4\sqrt{1 - h^{-4}\,\dot{r}^2}\right], \quad (2.3)$$

whose integrand is proportional to the proper length of a relativistic particle traveling in the warped space. Another familiar limit is the non-relativistic limit where $|h^{-4}g^{\mu\nu}\partial_\mu r\partial_\nu r| \ll 1$. The action then reduces to the minimal case

$$-T_3\int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu r\partial_\nu r + V(r)\right]. \quad (2.4)$$

As we will see, in terms of a D3-brane moving in extra dimensions, the action (2.2) comes from the DBI and Chern-Simons action describing the low-energy effective world-volume field theory of a probe brane in the AdS and R-R fields background.

We assume that the potential $V(r)$ has a maximum at $r = 0$ and falls as $r > 0$. For a generic non-flat potential, in the familiar case of (2.4), the scalar will undergo a fast-roll and make the inflation impossible. Here the highly warped space near $r \sim 0$ plays an important role. The idea is that the scalar velocity is restricted by the speed of light in the internal space $\dot{r} \leq h^2$. Therefore the requirement of slow rolling translates into the requirement of a small warp factor. This is interesting since an exponentially large warping is not difficult to find. In fact, it turns out that there are more stringent constraints coming from, for example, the strength of the background which supports the warp factor against the inflaton back-reaction, and the infrared closed string creation of the dS back-reaction. These constraints will be discussed in section 2.1 and section 3.1.

2.1 The inflation

We first study the zero-mode dynamics of the scalar inflaton, which drives the inflation. We ignore the spatial inhomogeneities of the scalar field so that it is only a function of time $r(t)$. The four-dimensional metric $g_{\mu\nu}$ is taken to be

$$\mathrm{diag}(-1, a^2(t), a^2(t), a^2(t)), \quad (2.5)$$

where $a(t)$ is the scale factor. The action (2.2) then becomes

$$S = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}R - T_3\int d^4x\; a^3(t)\left[h^4\sqrt{1 - h^{-4}\dot{r}^2} - h^4 + V(r)\right]. \quad (2.6)$$

The corresponding equations of motion are

$$3\left(\frac{\dot{a}}{a}\right)^2 = \frac{T_3}{M_{\rm Pl}^2}\left(\frac{h^6}{\sqrt{h^4-\dot{r}^2}} - h^4 + V\right), \tag{2.7}$$

$$\frac{d}{dt}\left(\frac{h^2\dot{r}}{\sqrt{h^4-\dot{r}^2}}\right) + 3H\frac{h^2\dot{r}}{\sqrt{h^4-\dot{r}^2}} + \frac{2h\partial_r h(2h^4-\dot{r}^2)}{\sqrt{h^4-\dot{r}^2}} - \partial_r\left(h^4 - V(r)\right) = 0\,. \tag{2.8}$$

We will only need the information of the potential near $r \sim 0$ and expand $V(r)$ as

$$V(r) \approx V_i - \frac{1}{2}m^2 r^2\,. \tag{2.9}$$

We will start the scalar inflaton deep in the warped space from $r_0 \sim 0$. A realization of such an initial condition will be provided in section 3.

For clarity, we make two approximations to be verified in the end of this subsection. First, we approximate that during inflation the potential $V(r)$ stays as a constant $V(0) \equiv V_i$. As we will see, this is because the inflaton moves over only a very short distance during the inflation. Second, the kinetic energy of the scalar field, namely the first two terms on the right hand side of the eq. (2.7), is negligible comparing to the potential V. This is because these two terms are red-shifted by the warped factor h^4. Both assumptions hold because during inflation the inflaton is held inside the IR region for a sufficiently long time. This will translate into a not-very-restrictive upper bound on V_i. The eq. (2.7) is then significantly simplified. It gives a dS space with a Hubble constant

$$H = \frac{\dot{a}}{a} = \frac{\sqrt{V_i T_3}}{\sqrt{3}M_{\rm Pl}}\,. \tag{2.10}$$

From now on we will denote $m^2 \equiv \beta H^2$ as long as H is a constant.

In the non-relativistic limit $\dot{r} \ll h^2$, the equation of motion (2.8) for r reduces to the familiar form

$$\ddot{r} + 3H\dot{r} + \partial_r V(r) = 0\,. \tag{2.11}$$

If $\beta \ll 1$, the potential (2.9) satisfies the slow-roll conditions and the eq. (2.11) determines the slow-roll velocity. It is also interesting to see how the warp factor will affect such dynamics and we study it in appendix B. Here we concentrate on the more general situation where $\beta \gtrsim 1$. In this case, the inflaton will be accelerated quickly to become relativistic if h^2 is small enough.

We thus expand the inflaton evolution around the speed of light

$$r = -\frac{R^2}{t}\left(1 - \frac{\lambda}{(-t)^p} + \cdots\right), \tag{2.12}$$

where we have chosen the time to run from $-\infty$. The leading contributions in eq. (2.8) come from the second term

$$\frac{3HR^2}{\sqrt{2(p-1)\lambda}\,(-t)^{2-\frac{p}{2}}} \tag{2.13}$$

and the potential term

$$\frac{\beta H^2 R^2}{t} \,. \tag{2.14}$$

The subleading terms are suppressed at least by a factor of $1/Ht$ and neglected if

$$t \ll -H^{-1} \qquad \text{or} \qquad r \ll R^2 H \,. \tag{2.15}$$

The parameters p and λ in (2.12) are determined by matching (2.13) and (2.14). We get

$$r = -\frac{R^2}{t} + \frac{9R^2}{2\beta^2 H^2}\frac{1}{t^3} + \cdots \,, \tag{2.16}$$

where the condition

$$t \ll -\beta^{-1} H^{-1} \qquad \text{or} \qquad r \ll \beta R^2 H \tag{2.17}$$

is required for such an expansion. For the case that we are mostly interested in, $\beta \gtrsim 1$, the condition (2.15) is stronger than (2.17).

As emphasized in ref. [17, 40], the back-reaction of the relativistic inflaton can have significant impact on the DBI action. The condition that such a back-reaction can be neglected can be estimated as follows. The warping scale caused by the energy density of the inflaton field in the internal space is characterized by $R'^4 \sim \gamma/T_3$, where $\gamma = h^2/\sqrt{h^4 - \dot{r}^2}$ is the Lorentz contraction factor. This scale has to be much smaller than that of the background R^4. Or equivalently, as we will discuss in section 3, the background warped space with $R^4 \sim N/T_3$ can be thought of as being created by N source D3-branes. The energy density of the relativistic probe D3-brane should be much smaller than the source for the back-reaction to be ignored. Using (2.16) this condition, $\gamma \ll N$, becomes

$$t \gg -3N\beta^{-1} H^{-1} \qquad \text{or} \qquad r \gg \frac{\beta R^2 H}{3N} \,. \tag{2.18}$$

Let us now summarize the dynamics of the inflaton inside the throat. Starting from the place $r \gg \frac{\beta R^2 H}{3N}$ where the back-reaction can be ignored, the inflaton travels ultra-relativistically toward the UV side of the warped space under the acceleration of the potential (2.9). The coordinate velocity is bound by the causality constraint and is very small. During this period, the inflaton is held up at the top of the potential for a sufficiently long time to trigger the inflation. The Lorentz contraction factor of the inflaton decreases in this process. Around $r \sim R^2 H$, the inflaton starts to become non-relativistic due to the increased warp factor. But the coordinate velocity is in fact much larger. Inflation is ended and the inflaton undergoes a fast-roll down to the bottom of the potential. During the whole inflationary period, the inflaton is relativistic. This period lasts for $\Delta t \sim N\beta^{-1} H^{-1}$, so the total number of inflationary e-folds is

$$N_{\rm tot} \approx H\Delta t \sim N\beta^{-1} \,. \tag{2.19}$$

To have a large-N, we need R to be bigger than $T_3^{-1/4}$. For example, if $R \sim 10 T_3^{-1/4}$, we have $N \sim 10^4$. In terms of string theory flux compactification, such a value is not difficult to find. In fact, as we will see from a more detailed model in section 5, a sufficient amount of inflation proceeds even if β is considerably larger than one. Within the range (2.15) and (2.18), the inflaton position r is related to the latest e-folds N_e by

$$N_e \approx HR^2/r \approx \frac{\sqrt{T_3}R}{\sqrt{3}M_{\rm Pl}}\sqrt{V_i}\; h^{-1}\,. \tag{2.20}$$

This expression can be turned around and viewed as a requirement for h in order to have N_e e-folds of inflation. This is easy to satisfy since the warp factor is usually exponentially small. Therefore we will consider the constraint from the back-reaction (2.19) to be stronger.

We have a few comments here.

Besides the lower bound (2.18) coming from the back-reaction, we will also have corrections related to the initial starting point r_0 at $-t_0$, if we assume the inflaton starts there with zero velocity. This gives the asymptotic behavior (2.16) a correction of order R^2/t_0.[1] Nonetheless as mentioned, because the main constraint from the back-reaction on the total number of inflationary e-folds is usually much stronger than the requirement of having a relatively large warping, we will always assume that the inflaton starts from a small enough r and the abovementioned correction can be ignored.

The motion of the inflaton within the region where the back-reaction cannot be ignored is under less precise control so far. A qualitative description is discussed in [40]. The time scale is expected to be roughly of order $N\beta^{-1}H^{-1}$ if we think of this region as having an effective warp factor similar to the lower bound (2.18). This will make the inflationary period last even longer. Since the total number of e-folds (2.19) is already very large, in this paper we assume this period to be outside of the observable universe.

More importantly, there are other more stringent constraints coming from the back-reaction of multiple D3-branes and infrared closed string creation. We will describe these in terms of strings and branes in section 3.1.

Interestingly, the DBI inflation persists even if $\beta \lesssim 1$. What happens is that, as we decrease β, a growing period of slow-roll inflation smoothly matches on to the end of a long period of DBI inflation. We will study this in appendix B.

We now check the consistency requirement for the two approximations made in the beginning of this subsection. First, the distance $\Delta r \sim R^2 H$ that the inflaton moves over during the inflation lowers the potential by $\beta H^4 R^4$. Second, in eq. (2.7), the kinetic energy is proportional to $h^4\gamma < R^4 H^4 \gamma \sim \beta N_e R^4 H^4$. Both are much less than V_i if

$$V_i T_3 \ll \frac{M_{\rm Pl}^4}{\beta N_e R^4 T_3}\,, \tag{2.21}$$

which is very easy to satisfy and normally having $HR \ll 1$ is enough.

[1] If $-t_0 \ll -\beta^2 N_e^3/H$, or $r_0 \ll R^2 H/\beta^2 N_e^3$, this correction does not affect the first two leading terms in (2.16), and therefore does not change our analyses. If $-\beta^2 N_e^3/H < -t_0 \ll -N_e/H$, or $R^2 H/\beta^2 N_e^3 < r_0 \ll R^2 H/N_e$, the second term in (2.16) is affected. But this will only lower the velocity and make the back-reaction smaller. Having larger $-t_0$ will then decrease the total number of e-folds.

2.2 Density perturbations

In the previous subsection we have studied the zero-mode evolution of the inflaton field and gravitational background. In this subsection we will study perturbations around it. In ref. [54], Garriga and Mukhanov have developed a general formalism to calculate the density perturbations for their kinetic energy driven inflation model [55]. Their analyses are very general and we can directly adapt them here as well. A similar application can be found in [24].

Before we start the rigorous derivation, we would like to present an intuitive approach [28] which gives a more explicit interpretation of the underlying physics in our case. As we have seen, a special property of the inflaton in our case is that it travels relativistically and the corresponding Lorentz contraction factor γ is decreasing. If we choose at each moment an instantaneous frame which moves at the same speed as the inflaton, the zero mode velocity of the inflaton vanishes to this observer. (It is a good approximation for large N_e. This is because $\Delta\gamma/\gamma \approx \Delta N_e/N_e$, so the relative change in γ is negligible in a duration of several e-folds.) Because of the time dilation, the Hubble constant is increased by a factor of γ to this moving observer. We can then use the result of the scalar fluctuations in the minimally coupled (non-relativistic) case, namely $\delta r' \equiv \delta\phi'/\sqrt{T_3} \approx H\gamma/(2\pi\sqrt{T_3})$. This amplitude is essentially determined by applying the uncertainty principle to the inflaton momentum generated within a Hubble horizon of size $H^{-1}\gamma^{-1}$. After they are stretched outside of the Hubble horizon, their amplitudes get frozen because they are no longer in causal contact. We then switch to the lab observer, the horizon size remains the same since it is in the direction orthogonal to the velocity. But the frozen scalar amplitude will be reduced by a factor of γ^{-1} because of the relativistic Lorentz contraction. So we get $\delta r \approx H/(2\pi\sqrt{T_3})$ which is the same as the slow-roll case, except that the horizon size is now reduced by a factor of γ^{-1}. This horizon is also called the sound horizon.

Because of these scalar inhomogeneities, different spatial part of the universe will end the inflation at different time [6, 7] (in a gauge where we set the unperturbed slice synchronous). For small perturbations $\delta r \ll r$, the time difference is

$$\delta t \approx \frac{\delta r_*}{\dot r_*} \approx \frac{H}{2\pi h_*^2\sqrt{T_3}} \approx \frac{N_e^2}{2\pi\sqrt{T_3}R^2 H}\,. \tag{2.22}$$

In the third step, eq. (2.20) is used. The subscript $*$ means that the variable is evaluated at the time of the horizon crossing when the corresponding mode is frozen. This time delay seeds the large scale structure formation [6, 7, 56, 57]. On the scale of Cosmic Microwave Background (CMB), the resulting density perturbation is given by

$$\delta_H = \frac{2}{5}\varepsilon_r H\delta t \approx \varepsilon_r \frac{N_e^2}{5\pi\sqrt{T_3}R^2}\,. \tag{2.23}$$

In the simplest case $\varepsilon_r = 1$. But for later purpose, we define $\varepsilon_r \equiv H_r\delta t_r/H\delta t$. Notice here that we have denoted the Hubble constant H_r during the reheating differently from the Hubble constant H during the inflation. In the usual field theory we normally assume that they are approximately equal. But applying to the multi-throat string compactification, they may be very different because the reheating can happen in a throat not responsible for the inflation. Independent warp factors cause the subtlety of a possible period of cosmological rescaling process in such a throat. This cannot be described by an effective single scalar field theory and may be imposed as a boundary condition. It also has the effect of shifting the wave-number and rescaling the $\delta t \to \delta t_r$ by a related factor. We leave these details specific to string models to sections 4 and 5.

Let us now start to apply the formalism from [54]. The fluctuations around the zero-mode evolution (2.5) and (2.16) can be parameterized in the following way [10]

$$\begin{aligned} ds_4^2 \equiv g_{\mu\nu}dx^\mu dx^\nu &= -(1+2\Phi)dt^2 + (1-2\Phi)a^2(t)d\mathbf{x}^2 \,, \\ r(t,\mathbf{x}) &= r_0(t) + \delta r(t,\mathbf{x}) \,, \end{aligned} \tag{2.24}$$

where we have added the subscript 0 to denote the zero-mode evolution. Following the notation in [54], we denote the pressure and energy density as

$$\begin{aligned} \frac{p(X,r)}{T_3} &\equiv -h^4\sqrt{1+2h^{-4}X} + h^4 - V \,, \\ \frac{\varepsilon(X,r)}{T_3} &\equiv \frac{h^4}{\sqrt{1+2h^{-4}X}} - h^4 + V \,, \end{aligned} \tag{2.25}$$

where

$$X \equiv \frac{1}{2} g^{\mu\nu}\partial_\mu r \partial_\nu r \,. \tag{2.26}$$

The equations of motion for the perturbations follow from the Einstein's equations

$$\frac{\partial}{\partial t}\frac{\delta r}{\dot r_0} = \Phi + \frac{2M_{\rm Pl}^2}{T_3}\frac{c_s^2}{\varepsilon+p}\frac{\nabla^2}{a^2}\Phi \,, \tag{2.27}$$

$$\frac{\partial}{\partial t}(a\Phi) = \frac{T_3}{2M_{\rm Pl}^2} a(\varepsilon+p)\frac{\delta r}{\dot r_0} \,, \tag{2.28}$$

where the sound speed c_s is defined as

$$c_s^2 \equiv 1 + 2h^{-4}X \,. \tag{2.29}$$

In (2.27) and (2.28), the ε, p and c_s are all evaluated by the zero-mode solutions, which are

$$\begin{aligned} c_s &= \frac{\sqrt{h_0^4 - \dot r_0^2}}{h_0^2} = \gamma^{-1} \,. \\ \frac{\varepsilon}{T_3} &= \gamma h_0^4 - h_0^4 + V(r_0) \,, \\ \frac{p}{T_3} &= -\gamma^{-1}h_0^4 + h_0^4 - V(r_0) \,, \end{aligned} \tag{2.30}$$

Using the new variables ξ and ζ,

$$\xi \equiv \frac{2M_{\rm Pl}^2 \Phi a}{T_3 H} \,, \tag{2.31}$$

$$\zeta \equiv H\frac{\delta r}{\dot r_0} + \Phi \,, \tag{2.32}$$

and the relation $\dot{H} = -\frac{T_3}{2M_{\rm Pl}^2}(\varepsilon + p)$, we can rewrite the equations of motion as

$$\dot{\xi} = \frac{a(\varepsilon+p)}{H^2}\zeta\,, \tag{2.33}$$

$$\dot{\zeta} = \frac{c_s^2 H^2}{a^3(\varepsilon+p)}\nabla^2\xi\,. \tag{2.34}$$

Further defining

$$\begin{aligned} z &\equiv \frac{a(\varepsilon+p)^{1/2}}{c_s H}T_3^{1/2}\,, \\ v &\equiv z\zeta\,, \end{aligned} \tag{2.35}$$

we can simplify eqs. (2.33) and (2.34) as

$$v'' - c_s^2\,\nabla^2 v - \frac{z''}{z}v = 0\,, \tag{2.36}$$

where the prime denotes the derivative with respect to the conformal time η defined by $d\eta = dt/a(t)$. Another equation is of first order and becomes auxiliary.

To evaluate z''/z we use (2.35) and (2.30). The leading contribution comes from the scale factor a which has the strongest time dependence. The next order comes from ε, p and c_s, which all vary more slowly. The time-dependence of H is neglected. So we get

$$\frac{z''}{z} \approx 2a^2H^2\left(1 + \frac{1}{2N_e} + \mathcal{O}\left(\frac{1}{N_e^2}\right)\right). \tag{2.37}$$

Hence for large N_e, eq. (2.36) reduces to the familiar equation that we encounter in the slow-roll inflation, except for the presence of the sound speed c_s. As usual, the solution can be obtained by matching the short wavelength behavior to the long wavelength behavior at the horizon crossing. For $k/a \gg H/c_s$,[2] the quantum fluctuations of v reduce to those in the flat space-time,[3]

$$v_k \approx \sqrt{\frac{1}{2c_s k}}\;e^{-ic_s k\eta}\,. \tag{2.38}$$

For $k/a \ll H/c_s$, it is also easy to get the solution

$$v_k \approx i\frac{H_*^2}{\sqrt{2}k^{3/2}h_*^2\sqrt{T_3}}\,z\,, \tag{2.39}$$

where the coefficient of z is obtained (up to a constant phase) by matching it to (2.38) at the horizon crossing $c_{s*}k = a_*H_*$ and using $z \approx ah^2c_s^{-3/2}H^{-1}\sqrt{T_3}$. Hence we see the well-known phenomenon that, in terms of ζ, the quantum fluctuations (2.38) evolves to the frozen classical perturbations (2.39). We also see that the horizon size is $c_{s*}H_*^{-1} = \gamma_*^{-1}H_*^{-1}$,

[2] The variation of c_s has to be small enough, $c_s'/c_s \ll kc_s/2\pi$. This is satisfied since $c_s'/c_s = a\dot{c}_s/c_s \approx aH/N_e \ll c_sk/N_e$.

[3] The Bunch-Davies vacuum is chosen here. For discussions on possible deviations from it, see e.g. [58] and references therein.

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agree with the previous intuitive argument. Under the assumption of instant and efficient reheating, the perturbations of the scalar field is transformed into density perturbations. The corresponding spectral density is then

$$\mathcal{P}_{\mathcal{R}}(k) \equiv \frac{1}{2\pi^2} k^3 |\zeta_k|^2 = \frac{H_*^4}{4\pi^2 h_*^4 T_3}\,, \tag{2.40}$$

where ζ_k is the Fourier mode of ζ defined in (2.32). The density perturbation δ_H is related to the spectral density $\mathcal{P}_{\mathcal{R}}(k)$ by

$$\delta_H^2 \equiv \frac{4}{25}\mathcal{P}_{\mathcal{R}}(k)\,. \tag{2.41}$$

So we recover the result (2.23) (except for a difference between $H_r \delta t_r$ and $H\delta t$ which we discuss below).

To compare with the previously mentioned physical interpretation, we obtain the relation between ζ and Φ using (2.28) and (2.32)

$$\begin{aligned} \zeta &= \frac{5\varepsilon + 3p}{3(\varepsilon + p)}\Phi + \frac{2\varepsilon}{3(\varepsilon + p)}\frac{\dot{\Phi}}{H} \\ &\approx \frac{2V}{3h^4\gamma}\left(\Phi + \frac{\dot{\Phi}}{H}\right). \end{aligned} \tag{2.42}$$

The second term is smaller than the first by a factor of $1/Ht \approx -1/N_e$. Since $V \gg h^4\gamma$, we have $\zeta \gg \Phi$. This means that the first term in (2.32) dominates. The physical interpretation of this term fits in our previous intuitive arguments in the convenient gauge choice. So a possible jump of the Hubble constant from H to H_r and the time delay from δt to δt_r, imposed as an approximate boundary condition at the reheating, is translated into a jump in ζ by a factor of ε_r. So the density perturbation will have an additional factor ε_r (as long as $\varepsilon_r \zeta \gg \Phi$). Such a mechanism is provided when we discuss more reheating details in section 4 and arises quite generally in some string models in section 5 and appendix B.

3. DBI inflation in string theory

It is important to ask how the field theory described in the previous section may be embedded in string theory. One natural place to realize it is to use the mobile D3-branes in the flux stabilized string compactification. This was described in a multi-throat brane inflation scenario [28]. In this setup, the position of branes in the extra dimensions is the inflaton as in the brane inflation [33], and the warped extra dimensions corresponds to the warped internal space.

Giddings, Kachru and Polchinski (GKP) [13, 12] show that, near a conifold singularity in type-IIB string compactification on a Calabi-Yau manifold, the presence of NS-NS and R-R three-form fluxes on two dual cycles induces the gravitational and R-R charges similar to those of the transverse D3-branes. The equivalent D3-charge is

$$N = MK\,, \tag{3.1}$$

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where K and M is the number of NS-NS and R-R fluxes respectively, and the characteristic length scale R of the resulting warped space is given by

$$R^4 = \frac{27}{4}\pi g_s N \alpha'^2 \,. \tag{3.2}$$

In addition, this warped space has a minimum warp factor in the IR end

$$h_{\rm min} \sim \exp\left(\frac{-2\pi K}{3Mg_s}\right) . \tag{3.3}$$

The fluxes generally fix the complex moduli and the axion-dilaton. A non-perturbative superpotential is used to stabilize the Kähler moduli and antibranes are introduced to lift the vacuum to dS space [14].[4]

A multi-throat configuration is a generalization of such a setup, which contains many throats of different warp factors in different places in the extra dimensions. It will be interesting to construct it explicitly, but in this paper we assume its existence. We add the D3-branes whose moduli in throats are the candidate inflatons. The volume stabilization for the extra dimensions and the interactions between the D3 and D7-branes will generate potentials for these D3-brane moduli. Details of such interactions are quite complicated and still under active studies. Specifically in this paper we will be interested in the following situation. Consider the situation where the D3-brane moduli receive quadratic potentials with mass-squared of $\mathcal{O}(H^2)$ or larger. This is actually a generic situation as we have seen in the eta-problem of the slow-roll inflation model building. In order to have slow-roll inflation, these contributions have to cancel each other to a certain precision. The tuning involved depends on the mass range of the contributing terms and adjustable parameters. Here we do not address the origin of these mass contributions. (Studies can be found in [16, 19, 25, 29, 30].) But we do not require the abovementioned fine-tuned cancellations. In the multi-throat setup, we assume some throats have negative mass-squared, and some have positive mass-squared. We note that these potentials are repulsive or attractive for the D3-brane moduli, but not the (fixed) positions of the throats.[5] The potential that we considered in (2.9) corresponds to those repulsive ones.

An immediate question is then how the D3-branes can start from the IR end of a repulsive throat. This can be done by considering the dynamics of anti-D3-branes in this setup. Like D3-branes, throats will attract and annihilate anti-D3-branes. This process undergoes through the flux-antibrane annihilation [39, 22]. However there are two important differences between the flux-antibrane annihilation and brane-antibrane annihilation. First, if the number p of the anti-D3-branes is much smaller than the R-R flux number M, this annihilation proceeds through quantum tunneling. So the anti-D3-branes in these throats can have different lifetime. This is necessary if antibranes are used to lift the AdS vacuum and provide the inflationary energy [14, 16]. Second, more important to our

[4]Alternatives are studied in [52, 53].

[5]Here we are only interested in throats located at various extrema of the D3-brane moduli potential profile. It is important to see that in what situation this can arise naturally (for example, for throats sitting at orbifold fixed points), or tuning has to be involved.

current discussion, a number of D3-branes will generally be created in the flux-antibrane annihilation. The reason is that when the anti-D3-branes annihilate against the NS-NS fluxes, the total D3-charge of the throat can only change in steps of M according to (3.1). Unless p is a multiple of M, D3-branes will be generated in the end of the annihilation to conserve the D3-charge. The moduli of these D3-branes become the inflaton required in our DBI inflation.

3.1 DBI action and its validity

The low energy world-volume dynamics of a probe D3-brane in a warped space such as (2.1) is described by the Dirac-Born-Infeld (DBI) and Chern-Simons action [59]

$$S = T_3 \int d^4\xi \left[-\sqrt{-\det\left(\partial_a X^M \partial_b X^N G_{MN}\right)} - \frac{1}{4!}\epsilon^{a_1 \cdots a_4} \partial_{a_1} X^{M_1} \cdots \partial_{a_4} X^{M_4} C_{M_1 \cdots M_4} \right]. \tag{3.4}$$

T_3 is the D3-brane tension, and ξ^a $(a = 0, 1, 2, 3)$ are the D3-brane world-volume coordinates. The functions $X^M(\xi^a)$ $(M = 0, 1, 2, 3, 4)$ describes the embedding of the D3-brane in the ambient space X^M, where $X^\mu \equiv x^\mu$ $(\mu = 0, 1, 2, 3)$ and $X^4 \equiv r$. (We ignore the other angular directions.) We are interested in branes transverse to the extra dimension r. In this case, the DBI action restricts the longitudinal scale of the brane to comove with the warped background [40]. Hence we can choose the convenient choice $x^\mu = \xi^\mu$ $(\mu = 0, 1, 2, 3)$ throughout the evolution (which is no longer true in section 4.2). We can then denote the embedding slice as $r(t, x^i)$, which can be regarded as a scalar field on the four-dimensional spacetime. This scalar describes the position and fluctuations of the brane in the transverse direction.

For D3-brane the R-R four-form potential is $C_{0123} = -\sqrt{-g}\, h^4(r)$, where the coupling $\sqrt{-g}$ is ensured by the four-dimensional Lorentz invariance with the convention $\epsilon^{0123} \equiv 1$.[6] With the addition of other possible potentials $V(r)$, the action (3.4) leads to (2.2). These additional potentials provide the inflationary energy. They can come from anti-D3-branes sitting inside the other throats, D3-D7 brane interactions, or related volume stabilization.

The validity of the DBI action requires that the energy involved in the effective field theory be much smaller than the mass of the massive W-bosons stretching between the probe brane and the horizon [59, 17]. From (2.16) this requirement, i.e. $\dot{r}/r \ll r/\alpha'$, becomes $R^2 \gg \alpha'$, which is in the region where we trust the supergravity background. More importantly, the probe dynamics is guaranteed only when the back-reaction of the D3-brane is small [17, 40]. This is the main constraint that we used in section 2.1. Now we can understand it more easily in this context. The warped space is the same as the near-horizon region of a stack of N D3-brane source. The relativistic effect increases the proper energy density of the probe D3-brane by a Lorentz contractor factor γ. If we treat the gravitational field strength of such a relativistic brane to be increased by roughly the same amount, we need $\gamma \ll N$ in order to neglect such a back-reaction.

There are other effects that are more restrictive than the lower bound (2.18). First, the number of D3-branes created by the flux-antibrane annihilation is $M - p$. If all these

[6] More explicitly, the four-form potential $C_{\mu_0 \cdots \mu_3} = \epsilon^{\nu_0 \cdots \nu_3} g_{\nu_0 \mu_0} \cdots g_{\nu_3 \mu_3} \frac{1}{\sqrt{-g}} h^4(r)$.

branes stick together and exit the throat, the back-reaction will be increased by a factor of M for $p \ll M$. So the total number of inflationary e-folds is reduced to

$$N_{\rm tot} \sim K\beta^{-1}\,, \tag{3.5}$$

which we approximate as $\sqrt{N}\beta^{-1}$.

Second, because the string scale is red-shifting towards the IR end, the Hubble expansion will be able to create closed strings some place in the throat. This is possible when the proper Hubble energy becomes comparable to the string scale, i.e. $h^{-4}H^4 \approx \alpha'^{-2}$. But in this subsection we are more interested in its effect on the background metric which is responsible for the brane speed limit. Such effect only gets significant when the energy density of the closed string gas/network becomes comparable to the source. It will then smear out the background metric and the effective warp factor will no longer decrease. Such a critical warp factor h_c can be estimated by $h_c^{-4}H^4 \sim NT_3$, where the left hand side is the proper energy density of the closed string gas/network smeared out in $r < r_c$, and the right hand side is the proper energy density of the source brane (or the equivalent fluxes). Using $R^4 \sim N/T_3$, we get $r_c \sim R^2H/\sqrt{N}$. So the total number of e-folds is reduced to $\sqrt{N}$.

So, for a stack (M) of branes, we will estimate $N_{\rm tot}$ as $\sqrt{N}\beta^{-1}$ from (3.5), while for a single brane, N_{tot} can be estimated as $\sqrt{N}$.

3.2 Stringy quantum fluctuations on D3-branes

According to section 2.2, the field quantum fluctuations on the D3-branes generate the density perturbations

$$\delta_H = \frac{2}{5}\varepsilon_r H\delta t \approx \varepsilon_r \frac{2N_e^2}{5\tilde{N}}\,, \qquad \tilde{N} \equiv \sqrt{\frac{27n_BN_B}{8}}\,, \tag{3.6}$$

where we have used (2.23), (3.2), $T_3 = (2\pi)^{-3}g_s^{-1}\alpha'^{-2}$ and considered n_B number of mobile D3-branes. The corresponding spectral index is

$$n_s - 1 \approx -\frac{4}{N_e}\,, \qquad \frac{dn_s}{d\ln k} \approx -\frac{4}{N_e^2}\,, \tag{3.7}$$

which is red and running negatively.

Red-shifted string scale also makes possible the open string creation on the mobile branes and modifies the field theory calculations of the density perturbations at some scale. This can be most easily seen by considering the following kinematic bound on the brane transverse fluctuations for the moving observer [28]. The quantum fluctuations are generated within a Hubble time and then get stretched out of the horizon. For the moving observer, the Hubble time is $\gamma^{-1}H^{-1}$. The longest distance that the brane fluctuations can travel in the transverse direction is then $\gamma^{-1}H^{-1}h^2$, where h^2 is the speed of light. In the field theory calculation (2.22), the fluctuation amplitude is $\gamma\delta r \approx \gamma h^2\delta t$, where we have restored the γ factor for the moving observer. It satisfies the kinematic bound only if

$$\gamma^2 H\delta t \lesssim 1\,. \tag{3.8}$$

This bound also has a dynamical interpretation. Using (2.22), eq. (3.8) is translated into

$$\gamma H \lesssim \sqrt{2\pi} T_3^{1/4} h \,. \tag{3.9}$$

This roughly means that the Hubble energy of the dS space has to be smaller than the red-shifted string scale, which is the valid region for field theories.

We note that the zero-mode field theory analyses should still remain valid, although the perturbation analyses break down beyond (3.8). As long as the background can be trusted under the conditions that we discussed in section 3.1, the only fact used for the zero-mode is the relativistic speed-limit constraint.

We can also rewrite (3.8) in terms of the latest e-folds N_e using $\gamma \approx \beta N_e/3$ [from (2.16)] and (3.6),

$$N_e \lesssim \sqrt{\frac{3}{\beta}} \tilde{N}^{1/4} \,. \tag{3.10}$$

Hence, comparing to the naive extension of (3.6) beyond (3.10), the bound (3.8) offers a suppression mechanism for larger scales. It is interesting that such a mechanism is built in without adding any extra features to the model.

Let us here simply suppose that for modes beyond (3.10) the bound is saturated and study some of its properties. The density perturbation is then

$$\delta_H \approx \varepsilon_r \frac{2}{5\gamma^2} \approx \varepsilon_r \frac{18}{5\beta^2 N_e^2} \,. \tag{3.11}$$

The spectral index,

$$n_s - 1 \approx \frac{4}{N_e} \,, \qquad \frac{dn_s}{d\ln k} \approx \frac{4}{N_e^2} \,, \tag{3.12}$$

is now blue and running positively. Also, (3.12) have to be smoothly connected to (3.7) through a transition region. Of course here we only studied the bound, and a full stringy treatment will be desirable to give more accurate account. Then we will have an interesting possibility to observe the stringy effects: branes, coming from an extremely infrared region (B-throat), imprint stringy information on their world-volume in terms of quantum fluctuations and bring them to our world (S or A-throat).

4. Throat reheating by relativistic branes

Reheating after inflation is important to populate the universe. In brane inflation, this is achieved by brane collision and annihilation in the S (or A) throat. In our model, this is sometimes caused by ultra-relativistic branes. In this section, we discuss two important processes for such a reheating [40], namely the relativistic collision and the cosmological rescaling.

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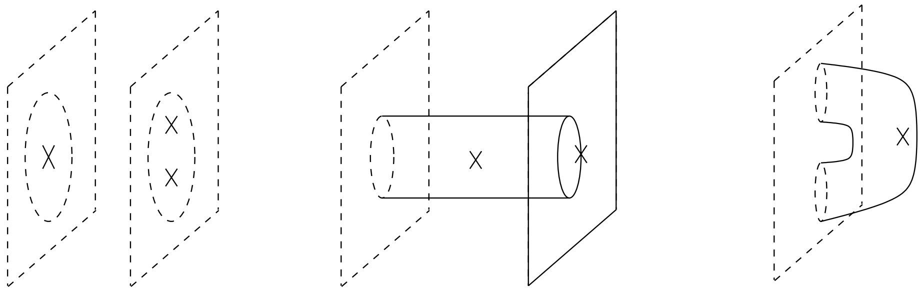

Figure 1: Closed and open string creation from brane annihilation. The branes in dashed lines are decaying ones, and the branes in solid lines are the surviving ones. (a) are some disk diagrams responsible for closed string creation. (b) is a loop diagram creating closed and/or open strings. The separation between the branes is in terms of the world-sheet distance, but branes may not be spatially separated. (c) is a loop diagram only creates closed strings.

4.1 Annihilation versus collision

We first discuss the direct string production in brane annihilation. The string dynamics in brane-antibrane annihilation is described by Sen's boundary conformal field theory of rolling tachyon [60 – 62]. Reference [63] has studied the one-point function on the disk diagram in this rolling tachyon background and show that it is capable of releasing all the brane energy to closed strings. What happens to the D3-anti-D3-branes is that the initial inhomogeneities on the brane world-volume will grow and eventually make them disconnected D0-anti-D0-branes, which then emit all the energy to a non-relativistic coherent state of heavy closed strings.

Since the Standard Model will have to live on some surviving D3-branes or anti-D3-branes, open string creation on such residue branes will be important for the Big Bang. Loop diagrams with one end on the rolling tachyon and another on the residue branes [64] then become interesting (see figure 1b). This is because the exponentially growing oscillator modes [65] in Sen's boundary state will create virtual closed strings and contribute to the loop diagrams. Due to their rapid time dependence, these are candidate competing diagrams against the disk and partially release brane energy to open strings. However, there are other loop diagrams with both ends on the rolling tachyon (see figure 1c). They only create closed strings. Although only a limit amount of information is known on such diagrams, it is not difficult to see that the evolution of 1c is much faster than 1b, since both ends of 1c are time-dependent while only one end of 1b is [64]. So again closed strings are dominantly produced in this process.[7] (It is possible that subsequently the heavy closed strings decay to both massless closed and open strings. This cosmological consequence deserves further studies [66].)

The annihilation process is important when the colliding branes and antibranes are non-relativistic. For example in KKLMMT, if we assume that the slow-roll conditions hold all the way from the UV entrance to IR end, the brane velocity will remain far below the

[7]We assume that the difference between the closed and open string couplings is not big.

speed limit. However in our case, there may not be a direct relation between the inflationary energy scale, which can come from a steep moduli potential, and the total warping of the S-throat. Hence the velocity of the D3-branes may be much faster and there may exist a region in the S-throat where the branes move relativistically. Such fast-rolling D3-branes will cause interesting effects on the reheating details.

The first feature is that the probe branes can become ultra-relativistic, and the maximum value of its Lorentz contraction factor is determined by the D3-charge of the background throat.

To illustrate, let us consider a quadratic attractive potential in the S-throat

$$V_S = \frac{1}{2} m_S^2 r^2 \,, \tag{4.1}$$

with a positive m_S^2 [see (A.8)]. Consider n D3-branes rolling out of the B-throat enter this S-throat directly. The total inflationary potential V_i in (2.9) is a net contribution of the repulsive potential (2.9) of the B-throat and the attractive potential (4.1) of the S-throat. After inflation, this potential is converted to the D3-brane kinetic energy when they are in the S-throat (but still away from the IR end). This provides the initial velocity for the D3-branes. We denote this velocity as v_0 and it is given by

$$\frac{1}{2} n v_0^2 \approx V_i \,. \tag{4.2}$$

A detailed dynamics of such D3-branes can be solved using the DBI action, and we can find the corresponding place where the probe back-reaction becomes important. We discuss this in more details in appendix. A. Here let us summarize the relevant main results.

It turns out that as long as the initial velocity v_0 satisfies

$$v_0 < \frac{M_{\rm Pl}^2}{T_3 R_S^2 (n N_S)^{1/2}} \,, \tag{4.3}$$

the gravitational coupling of these probe branes can be ignored. The resulting dynamics then becomes very simple. It is determined by the conserved energy density

$$\mathcal{E}/n = \frac{h^6}{\sqrt{h^4 - \dot{r}^2}} - h^4 + V_{\rm net}(r) \,. \tag{4.4}$$

The D3-branes go through three different phases after the inflation. In the first stage they are non-relativistic and accelerated by the potential (2.9) and (4.1) (mainly in the UV sides of the B and S-throat) to reach a velocity v_0. Such a velocity reaches the speed-limit at $h \approx \sqrt{v_0}$ in the S-throat and the branes enter the second relativistic phase. During this phase the energy density (4.4) is dominated by the first term, i.e. the kinetic energy. The proper spatial volume of the branes shrinks and the conserved coordinate energy density is converting from the brane tension to the relativistic kinetic energy. [This does not happen in the non-relativistic phase although the proper spatial volume is also shrinking because of the cancellation from the R-R field, which is the second term in (4.4).] The Lorentz contraction factor is increasing as $\gamma \approx \frac{1}{2} v_0^2 h^{-4}$. At

$$h_r \approx \frac{\sqrt{v_0}\, n^{1/4}}{N_S^{1/4}} \,, \tag{4.5}$$

$n\gamma$ becomes $\mathcal{O}(N_S)$ and the probe back-reaction becomes important. The D3-branes then enter a non-comoving phase. We will have more to say about this phase in the next subsection.

As long as the reheating happens after the first phase, the energy transfer is dominated by relativistic collision rather than annihilation. In terms of direct open string creation, this process does not have the abovementioned problem associated with the brane annihilation. Namely, in figure 1, regarding both branes as colliding ones, the interaction between the colliding branes is only in terms of diagrams like (B). We will estimate the energy density of the created open strings to be in the same order of magnitude as the collision energy density. Some interesting properties of the relativistic brane collision are studied in [67].

4.2 Cosmological rescaling

We now discuss the second effect closely related in the same process. If the reheating happens during the second phase discussed above, the reheating energy density is still approximately the same as the inflationary energy density, as in the non-relativistic annihilation case.[8] It is only the way of energy transfer that has been changed from the annihilation to ultra-relativistic collision (which is good in terms of direct open string creation). However, this is no longer true if the reheating happens in the third non-comoving phase. We will argue that such a phase will introduce effects not captured in an effective field theory, for example, a jump in the Hubble constant.

Although the precise mathematical description of the brane dynamics when back-reaction is significant is unavailable, we can think of an analogy of two identical stacks of branes approaching to each other. Because their energy density are similar, one will not feel the space being exponentially warped by another. Therefore the longitudinal scale of the brane does not significantly contract. A similar phenomenon for the relativistic branes should also happen. Where this takes place is taken to be at h_r given in (4.5), where the energy density of the relativistic probe branes is comparable to the source branes (or the equivalent fluxes). Starting from h_r, the warped background becomes negligible to those probe branes, and their proper volume is no longer contracting significantly.

Once these branes collide with other branes at the IR end, they will oscillate and expand. Their energy density is decreasing through expansion or radiation. In the mean while, the background is restoring. In fact it does not take too much expansion to reduce the energy density of these heated branes, say to one tenth of the original value. After that, they can again be treated as a probe of the background. Since we want this process to be connected to the standard Big Bang, we will be interested in the Poincare observer on the D3-branes. To this observer, in the end of the restoration process, the Planck mass takes the usual value in the sense of Randall and Sundrum. This coordinate choice of such an IR Poincare observer is important, the scale of such a choice is indicated in figure 2 by a dashed brane. (The proper energy is independent of such a choice.) To this observer the space-time inhomogeneity scale on the probe D3-branes has changed. This is illustrated in

[8] For annihilation this is true if we assume that the energy transfer to open strings during the reheating is rapid and efficient.

figure 2. These inhomogeneities have been geometrically rescaled by a factor of $g_r = h_r/h_S$, where h_S is the total warping of the S-throat. In the previous example,

$$g_r \sim \frac{n^{1/4} v_0^{1/2}}{N_S^{1/4} h_S} \,. \tag{4.6}$$

To obtain an order of magnitude estimate, we will ignore the fast restoration process and simply apply the rescaling factors of g_r to the corresponding length scale, time duration, or energy scale with respect to their values at h_r. These effects, not described in a scalar field theory, are then approximated as imposing effective boundary conditions in the beginning of the reheating. For example, the time difference δt of the inflation ending is geometrically increased by a factor of g_r; the Hubble constant is reduced by a factor of g_r^{-2} because the energy density is geometrically decreased by a factor of g_r^{-4}. Such rescaling effects can reduce the ζ in (2.32), and therefore the density perturbations, as we will see in an example of the next section.

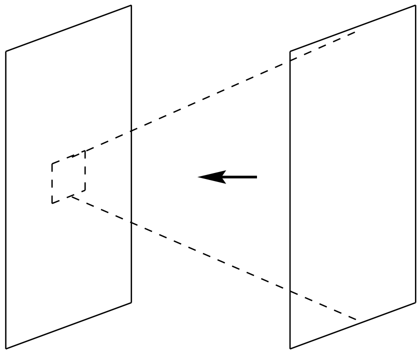

Figure 2: The cosmological rescaling for branes in the non-comoving region where back-reaction is large. The brane in dashed lines in the IR side indicates its longitudinal scale if it had followed the DBI action, which is the scale of the Poincare observer after the background restoration.

5. A multi-throat model

A multi-throat brane inflationary scenario has been described in [28], also in the introduction and section 3. So here we only briefly summarize some of the main points. We start by looking at the anti-D3-branes in the multi-throat configuration. They are attracted toward the throats, either annihilate against the fluxes through classical process, or stay inside in a quasi-stable state and annihilate through quantum tunneling. The end products are generally some D3-branes. For those throats (B-throats) having potentials like (2.9) for the D3-brane moduli, D3-branes will exit. The DBI inflation discussed in sections 2 and 3 then takes place. These branes eventually settle down in throats (S or A) with attractive D3-brane moduli potential, or in the bulk. The purpose of this section is to make this model more quantitative and improve the calculations by taking into account the aspects described in sections 3.2 and 4. We also discuss the tension of the cosmic strings created at the end of the inflation from brane annihilation/relativistic collision, and during the Hagedorn transition of the dS epoch.

We first consider two-throat case with only B and S-throats, where the S-throat is defined to have a RS warp factor. The Hubble constant is simply

$$H = \frac{\sqrt{V_i T_3}}{\sqrt{3} M_{\rm Pl}} \,. \tag{5.1}$$

The inflationary potential can be dominantly provided by the antibranes in the S-throat, or a moduli potential. This will be discussed in Case A and Case B, respectively.

The initial velocity v_0 of n number of D3-branes entering the S-throat is determined by the moduli potential. In Case A we have

$$\frac{1}{2}nv_0^2 < V_i \approx 2n_S h_S^4 \,. \tag{5.2}$$

In Case B we have

$$\frac{1}{2}nv_0^2 \approx V_i \,. \tag{5.3}$$

In the latter case, the velocity does not have to be as small as in the former, because the height of the moduli potential is not related to the S-throat warp factor. Then the rescaling will generally happen in a deep throat. In Case C, we consider the addition of an A-throat, where antibranes there are the main source of V_i.

Case A. If the reheating process involving brane collision and/or annihilation happens before the DBI action breaks down in the S-throat, we have the usual relation $H \approx H_r$ (assuming an efficient reheating to open strings). Such a situation happens when the initial velocity of the brane is small so that

$$h_S > h_r \,, \tag{5.4}$$

where h_r is given in (4.5). For example, if the inflationary energy is dominated by n_S antibranes at the end of the S-throat, we have $V_i \approx 2n_S h_S^4$. The D3-brane kinetic energy density ($\sim \frac{1}{2}nv_0^2$) has to be smaller than V_i since the moduli potential is not dominant. Hence the condition (5.4) is satisfied. In such a case, we notice that the density perturbation

$$\delta_H \approx \frac{2N_e^2}{5\tilde{N}} \tag{5.5}$$

is independent of the warp factor (and V), so h_S can take e.g. e^{-30} to incorporate the RS model. However at the same time fitting the observation $\delta_H \approx 1.9 \times 10^{-5}$ requires a very large N_B [31, 28] (similar to [24]). We take $N_e \approx 32$, because the inflation is driven by the electro-weak scale of the S-throat, and get $N_B \sim 2.4 \times 10^9$ (estimating $n_B \sim \sqrt{N_B}$). The total number of inflationary e-folds is 5×10^4 for $\beta \approx 1$. If we require the stringy suppression for δ_H discussed in section 3.2 happens near the largest observable scales, e.g. $(N_e)_c \approx 29$, we need $\beta \approx 15$. Then the total e-folds becomes 3×10^3.

This is our simplest case, but it remains to be seen how naturally we can get such a large N_B. (Getting a large N_B through orbifolding is discussed in [24].) It is interesting to note that N_B can be significantly reduced if the density perturbations are seeded by (3.11) as we shall discuss more in the later comments. In the next, we will discuss the case where the possible cosmological rescaling helps to reduce the density perturbations, and therefore N_B.

Case B. As we emphasized, in our scenario, the inflationary energy does not have to be correlated with the warp factor of the S-throat. It can also be sourced by the steep moduli potential. Then the reheating Hubble constant H_r is different from H if the reheating happens in the non-comoving region in the S-throat. It is determined by the energy density V_r on the reheated branes after the background restoration. This can be estimated following the description of section 4.2,

$$V_r \approx \alpha \cdot \frac{1}{2} n v_0^2 \cdot g_r^{-4} \approx \alpha N_S h_S^4 \,. \tag{5.6}$$

In the first step, we approximate the first factor $\alpha \sim 0.1$, that is, assume that the D3-branes can be treated again as a probe when their energy density is reduced to one tenth of the source. Reasonable variation of α will not significantly affect our later estimates. The second factor is the conserved coordinate energy density of the relativistic branes in the comoving region. The last factor is the rescaling factor. The result can also be simply understood as follows. As long as branes enter the non-comoving region, the final Lorentz contraction factor N_S is determined by the strength of the background and is independent of the initial brane velocity or the place where the DBI action breaks down. The reheating Hubble constant is now

$$H_r \approx \frac{\sqrt{\alpha N_S T_3} h_S^2}{\sqrt{3} M_{\rm Pl}} \,. \tag{5.7}$$

The reheating time delay after the rescaling process is

$$\delta t_r \sim g_r \delta t \approx \frac{g_r N_e^2}{H \tilde{N}} \,, \qquad \tilde{N} \equiv (27 n_B N_B/8)^{1/2} \,, \tag{5.8}$$

where eqs. (3.6) is used. The density perturbation can be estimated as

$$\delta_H \approx H_r \delta t_r \sim \frac{g_r \alpha^{1/2} N_S^{1/2} N_e^2 h_S^2}{\tilde{N} V_i^{1/2}} \,. \tag{5.9}$$

$$\sim \frac{\alpha^{1/2} N_S^{1/4} h_S}{V_i^{1/4}} \frac{N_e^2}{\tilde{N}} \,. \tag{5.10}$$

In the last step, eqs. (4.6) and (5.3) are used. The first factor in (5.10) is the effect of the rescaling. At $h_S \sim h_r$ (and $\alpha \sim 1$), it smoothly goes to one and we recover (5.5).

We turn eq. (5.10) around and use the measured density perturbation at the corresponding e-fold to determine the responsible inflationary potential,

$$V_i^{1/4} \sim \frac{\alpha^{1/2} N_S^{1/4} h_S N_e^2}{\delta_H \tilde{N}} \,. \tag{5.11}$$

As we discussed in section 3.2, there is a natural suppression mechanism for the density perturbations at long wavelengths. This happens at the critical e-folding

$$(N_e)_c \approx \sqrt{\frac{3}{\beta}} \, \tilde{N}^{1/4} \,. \tag{5.12}$$

If this is responsible for the observed CMB suppression near the IR end, we can determined $\tilde{N}$. This is the strategy that we will use in the following to determine the values of $\tilde{N}$ and N_e.

To do this we first estimate the total number of e-folds needed to account for the observable universe. We focus on the largest scale $R_0 \approx 10^{42}\,\mathrm{GeV}^{-1} \approx 10^4$ Mpc near the IR end of the CMB. The corresponding scale R_r at the time of the reheating can be estimated by the relation

$$R_r \approx \frac{T_0}{T_r} R_0 \,, \tag{5.13}$$

where $T_0 \approx 2.7$ K is the current temperature and the reheating temperature $T_r \approx V_r^{1/4}$. On the other hand, the Hubble length after rescaling is

$$(l_H)_r \equiv g_r l_H \sim g_r \gamma^{-1} \frac{M_{\rm Pl}}{\sqrt{V_i T_3}} \,. \tag{5.14}$$

Here the factor g_r is the rescaling effect discussed in section 4.2. The factor γ^{-1}, also known as the sound speed, is the relativistic effect discussed in section 2.2. γ can be calculated using (2.16), $\gamma \approx \beta N_e/3$.

Equations (5.11), (5.13) and (5.14) tell us the e-fold corresponding to the IR end of the CMB,

$$N_e = \ln \frac{R_r}{(l_H)_r} \approx 78 + \ln \frac{T_3^{1/4} h_S}{M_{\rm Pl}} + 3 \ln N_e + \ln \frac{\beta \alpha^{1/4} N_S^{1/4}}{\tilde{N}} \,. \tag{5.15}$$

$\tilde{N}$ and N_e can be determined by requiring that the $(N_e)_c$ in (5.12) is several (e.g. three) e-folds below the N_e in (5.15) (setting $\beta \approx 1$ here). Therefore the inflationary energy scale (5.11) largely depends on the IR scale $h_S T_3^{1/4}$ of the S-throat.

If we assume that the S-throat solves the hierarchy problem according to Randall and Sundrum by setting the IR scale $h_S T_3^{1/4}$ to be around TeV, this determines the N_e and $(V_i T_3)^{1/4}$ regardless of the actual value of T_3. We get

$$N_B \sim 8 \times 10^6 \,, \qquad n_B \sim \sqrt{N_B} \,, \qquad \tilde{N} \sim 2.8 \times 10^5 \,,$$
$$(N_e)_c \approx 40 \,, \qquad N_e \approx 43 \,, \qquad g_r \sim e^5 \,, \tag{5.16}$$
$$(V_i T_3)^{1/4} \sim 10^6\,\mathrm{GeV} \,. \tag{5.17}$$

In this estimation, we used $\alpha \sim 0.1$, $\beta \sim 1$ and $N_S \sim 10^4$. Among the 48 e-folds of the horizon stretching required to account for the homogeneity and flatness of our observable universe, 43 e-folds is given by the inflation, and the last five e-folds is given by the rescaling. The total number of inflationary e-folds is $N_{\rm tot} \sim \sqrt{N_B}/\beta \sim 3 \times 10^3$ for $\beta \sim 1$.

Case C. There are other possibilities. Let us consider adding an A-throat and the inflation ends by brane-antibrane annihilation in this throat. One option is to assume that the hierarchy problem is not or only partially solved by the RS mechanism, e.g. we live in an A-throat. Then for example in case A (replacing the subscript S with A), $h_A T_3^{1/4}$ needs not be TeV. From eq. (2.20), the N_e e-folds of inflation happens as long as h_A and h_B satisfy the relative relation [28]

$$\frac{h_B}{h_A^2} \lesssim \sqrt{\frac{2 n_A}{3}} \frac{R_B \sqrt{T_3}}{M_{\rm Pl} N_e} \,. \tag{5.18}$$

Note that this includes the case where the only throat required is the B-throat, namely $h_A = 1$. The upper bound on the inflation scale comes from the current experimental tensor modes bound [68, 69], $\mathcal{P}^h/\mathcal{P}_{\mathcal{R}} = \frac{8}{75\pi^2}\frac{V_i T_3}{\delta_H^2 M_{\rm Pl}^4} < 0.5$, which gives

$$V_i T_3/M_{\rm Pl}^4 < 1.7 \times 10^{-8}\,. \tag{5.19}$$

Another interesting option is to further add an S-throat. Then the reheating may happen either because some branes coming out of the B-throat enter both the A and S-throats [28, 31], or the KK modes of the decay products in the A-throat are transfered to the S-throat [66]. In either case, all branes in the A-throat have to be annihilated, and the fate of closed strings in the A-throat or how much magnitude of density perturbations can be transfered from A to S deserve further investigations [66].

We have a few additional comments on various aspects of this model.

Parameter dependence. There are some uncertainties in the estimations: for example in case B, the detailed rescaling and background restoration process parameterized by α in (5.6), and the steepness of the D3-brane moduli potential parameterized by β. The former only weakly affects (5.11) and (5.15). The variation of β changes the total number of e-folds. More importantly it changes the $(N_e)_c$ in (5.12). The spectral index (3.7) [and (3.12)] does not depend on the overall variation of δ_H that we talked about.

Tensor modes. In our model the condition for the inflation to happen is not restricted by the inflationary energy scale. So the tensor modes bound is very easy to satisfy. For example in Case B, $H/M_{\rm Pl} \sim 10^{-25}$; Case C is more flexible. We leave non-gaussianity feature for future studies.

Large N_B. For example in Case B, N_B is 8×10^6. This requires the NS-NS and R-R flux number to be a few thousands. In GKP compactification, the total D3 charge of all throats and (anti)branes equals to $\chi/24$, where χ is the Euler number of the corresponding fourfold in F-theory. The largest value we know of is $\chi/24 = 75852$ [70, 71]. It is so far not clear if we have an actual maximum value, but it seems that the model described in this section works if the Euler number is on the larger side.

We should emphasize here that the large N_B in Case B is no longer due to the fitting of δ_H. After taking into account of the non-comoving rescaling, such a degree of freedom goes to the factor V_i in eq. (5.10). N_B is large because we want to calculate most part of the density perturbations for our observable universe by the conventional field theory, and only invoke the open string quantum fluctuations at the early epoch as a mechanism to suppress the IR end of CMB. The relation (5.12) typically gives a large N_B.

However it remains an open possibility that considerable part of the universe is seeded by stringy quantum fluctuations. If this is the case, the only constraint on N_B is $\sqrt{N_B}/\beta \gtrsim N_e$ for $\sqrt{N_B}$ number of branes, or $\sqrt{N_B} > N_e$ for a single brane. Using (3.11), the density perturbations $\delta_H \approx 1.9 \times 10^{-5}$ is fit around $N_e \approx 55$ by choosing $\beta \approx 8$ ($\varepsilon_r = 1$). We only require $N_B \gtrsim 2 \times 10^5$, (or $N_B \gtrsim 3 \times 10^3$ for a single brane), which amounts to a few hundreds (or tens) of flux numbers.

String statistics. There is another interesting angle to look at the large N_B aspect. If the underlying parameters used to determine the total number of inflationary e-folds is not directly correlated with the degrees of freedom used for the string statistics [71], these two subjects can be independent of each other. However in our model we have seen a clear relation between the total e-folds and flux number, $N_{\rm tot} \sim \sqrt{N_B}$ for $\beta \approx 1$. Such a large factor adds a significant weight to the statistics and may push the selection rule to favor a compactification with a large Euler number.

In fact, we expect the same qualitative argument to apply to the slow-roll case as well, since more fluxes provide more degrees of freedom to adjust the shape of the potential, although the relation is much less explicit.

Cosmic strings. The brane inflation gives interesting mechanism to produce cosmic strings after inflation [41 − 43]. The D or F-strings are produced during brane-antibrane annihilation through Higgs mechanism or confinement. Their properties are further studied in [44 − 51]. The situation for the relativistic brane collision is similar, because right after collision the gauge symmetry is also restored due to high energy density transfered from the relativistic branes. In the subsequent cosmological evolution, this string network approaches the attractor scaling solution, and can be observed if their tension is large enough. For strings in the A-throat, the tension is given by (e.g. F-strings)

$$G\mu_F = \sqrt{\frac{g_s}{32\pi}} \frac{T_3^{1/2} h_A^2}{M_{\rm Pl}^2} \,. \tag{5.20}$$

Because strings will be generated if the temperature reaches the Hagedorn temperature [72, 43], there is another interesting way that strings can be produced in any inflationary model in the multi-throat configuration. Same as we discussed in the end of section 3.1, Hubble energy can exceed the string scale in a deep throat. We take such a relation to be

$$\frac{H}{2\pi} \gtrsim \frac{h}{4\pi\alpha'^{1/2}} \,, \tag{5.21}$$

where the left hand side is the Gibbons-Hawking temperature and the right hand side is the red-shifted Hagedorn temperature. Therefore any throat with a warp factor

$$h \lesssim \left(\frac{2}{9\pi^3}\right)^{1/4} \frac{\sqrt{V_i} T_3^{1/4}}{g_s^{1/4} M_{\rm Pl}} \tag{5.22}$$

is filled with strings. After inflation, strings will stay inside each of these throats by gravitational attraction and evolve. Hence each throat independently contributes to the string network. The corresponding string tension is

$$G\mu_F \lesssim \frac{1}{12\pi} \frac{V_i T_3}{M_{\rm Pl}^4} \,. \tag{5.23}$$

This is typically lower than (5.20).

If observable, we have a very clear signal for the multi-throat compactification — an isolated spectrum of the highest string tension (5.20), whose existence depends on the existence of the A-throat, followed by a dense spectrum of lower tension (5.23). The string tension is associated with the inflation scale, so in Case A and B they are too weak to be observed. In Case C, the bound (5.19) gives the A-throat string tension

$$G\mu_A < 9 \times 10^{-6} \sqrt{\frac{g_s}{n_A}} , \tag{5.24}$$

which may be observed by future experiments, and the Hagedorn string tension

$$G\mu_H < 4 \times 10^{-10} . \tag{5.25}$$

Although the tension is much weaker, the latter can be enhanced by effects of the multiple throats (e.g. more signals).

Such a spectrum also arises in a multi-throat slow-roll model that we will discuss in appendix B. There we will also have a significant lower bound on $G\mu_A$.

6. Concluding remarks

In this paper we have studied a DBI inflation model in both field and string theory as an alternative to the slow-roll inflation. The inflation in such a model is achieved by a inflaton field held on the top of a steep potential by the IR end of a warped space.

This model is realized in the multi-throat string compactification setup. We demonstrate that, at least at the level of orders of magnitude, this model can simultaneously produce many interesting features. This includes the large number of inflationary e-folds; density perturbations of the right order of magnitude while incorporating the Randall-Sundrum model with a direct reheating; a scale-invariant spectrum with interesting features; and a possible mechanism for the infrared suppression on CMB.

Many issues remain to be studied in more details. For example, more detailed information on moduli potential profiles of the multi-throat GKP compactification with D3-branes; the stringy quantum fluctuations at the early stage; the detailed analysis of the non-gaussian feature (in some cases properly taking into account the rescaling or stringy effects); various back-reactions such as the closed string creation in the IR B-throat due to the dS back-reaction, and the rescaling process during the reheating due to the probe brane back-reaction; and some global aspects of this scenario such as its genericness and eternity.

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A. Rolling branes in S-throat

In this appendix, we are interested in how n number of D3-branes roll in the S-throat according to the DBI action. Most of the results can be found in refs. [17, 24, 40]. The main results have been used in section 4.

This dynamics is described by the equations of motion (2.7) and (2.8), but the potential $V(r)$ is replaced by

$$V(r) = \frac{1}{2} m_S^2 r^2 \,. \tag{A.1}$$

Here $m_S \gtrsim H$ so gives a generic steep attractive potential. The region that we will be interested in is well within the S-throat. We start with the initial velocity $-v_0$. Such a velocity is picked up after the D3-branes fall down the potentials (2.9) and most of (A.1). We first check that this velocity does not change much by the Hubble friction term if the warp factor $h \gg \sqrt{v_0}$.

The Hubble constant is mainly given by the kinetic energy of the non-relativistic brane after it falls down the potential,

$$H \approx \frac{\sqrt{nT_3}}{\sqrt{6} M_{\rm Pl}} v_0 \,. \tag{A.2}$$

The velocity change due to the second Hubble friction term in (2.11) can be estimated as follows

$$\Delta \dot{r} \sim \frac{\sqrt{nT_3}}{M_{\rm Pl}} v_0^2 \Delta t < \frac{\sqrt{nT_3} R_S}{M_{\rm Pl}} v_0 \,. \tag{A.3}$$

Since we can estimate $\sqrt{nT_3} R_S / M_{\rm Pl} \sim N_S^{1/4} n^{1/2} T_3^{1/4} / M_{\rm Pl}$, which is typically not much bigger than one, the brane velocity remains in the same order of magnitude.

Because of the decreasing warp factor, this velocity reaches the speed-limit around $h \sim \sqrt{v_0}$. We then return to the eq. (2.8). The factor of H in the second term is still given by the conserved (mostly kinetic) energy (A.2) if this term is negligible to the first term. This amounts to a comparison between H and $d/dt \sim 1/t$. If $t \ll M_{\rm Pl}/(\sqrt{nT_3} v_0)$, the gravitational coupling in (2.8) can indeed be ignored. We have a conserved coordinate energy density (4.4) with $\mathcal{E} \approx n v_0^2/2$ ($V_{\rm net}$ is approximately zero in the IR side of the S-throat). The D3-branes become relativistic

$$r = \frac{R_S^2}{t} - \frac{R_S^{10}}{7 v_0^2 t^9} + \cdots \,. \tag{A.4}$$

Now the time coordinate is chosen to be positive. If $t \gg M_{\rm Pl}/(\sqrt{nT_3}v_0)$, we need solve (2.7) and (2.8), and get

$$\begin{aligned} r &= \frac{R_S^2}{t} - \frac{\alpha R_S^2}{t^5} + \cdots\,, \\ a &\propto t^p\,, \end{aligned} \tag{A.5}$$

where

$$\alpha = \frac{\left(-1 + \sqrt{1 + 3nm_S^2 T_3 R_S^4/2M_{\rm Pl}^2}\right)^2}{6m_S^4}\,, \tag{A.6}$$

$$p = \frac{nT_3 R_S^4}{2M_{\rm Pl}^2\sqrt{6\alpha}}\,. \tag{A.7}$$

Let us first consider the case

$$m_S^2 \ll M_{\rm Pl}^2/(nT_3R_S^4) \sim \frac{M_{\rm Pl}^2}{(nN_S)}\,, \tag{A.8}$$

where these parameters are simplified,

$$\alpha \approx \frac{3(nT_3)^2 R_S^8}{32M_{\rm Pl}^4}\,, \qquad p \approx \frac{2}{3}\,. \tag{A.9}$$

In this case the slow expansion of the scale factor $a(t)$ is driven by the kinetic energy of the D3-branes. One can check that the second terms of (A.4) and (A.5) match each other at the turning point $t \sim M_{\rm Pl}/(\sqrt{nT_3}v_0)$.

We now summarize the D3-brane dynamics following the DBI action. For $h \gg \sqrt{v_0}$, the brane is moving non-relativistically. Within $v_0 R_S\sqrt{nT_3}/M_{\rm Pl} \lesssim h \lesssim \sqrt{v_0}$, the brane travels relativistically. The gravitation is approximately decoupled in this region, and the brane dynamics (A.4) is determined by a conserved coordinate energy density (4.4). In this period, the Lorentz contraction factor increases as $\gamma \approx \frac{1}{2}v_0^2 h^{-4}$. For $h \lesssim v_0 R_S\sqrt{nT_3}/M_{\rm Pl}$, the gravitation coupling becomes important. The kinetic energy of the moving brane drives the expansion of the scale factor as in (A.5). The Lorentz contraction factor increases as $\gamma \approx h^{-2}M_{\rm Pl}^2/(nT_3R_S^2)$.

The probe back-reaction is important if $n\gamma \sim N_S$. For $v_0 < M_{\rm Pl}^2/T_3R_S^2(nN_S)^{1/2}$, it happens within the second interval. This is what we used in section 4.

We now consider the large mass-squared

$$m_S^2 \gg \frac{M_{\rm Pl}^2}{(nT_3R_S^4)} \sim \frac{M_{\rm Pl}^2}{nN_S}\,. \tag{A.10}$$

In this case, refs. [17, 24] show that the inflation is possible. Now the spatial expansion is driven by the potential energy of the branes. In (A.5), we have

$$\alpha \approx \frac{nT_3R_S^4}{4m_S^2 M_{\rm Pl}^2}\,, \qquad p \approx \frac{m_S\sqrt{nT_3R_S^4}}{\sqrt{6}M_{\rm Pl}}\,. \tag{A.11}$$

The probe back-reaction restricts

$$h > \frac{n^{1/4} m_S^{1/2} M_{\rm Pl}^{1/2}}{N_S^{1/2} T_3^{1/4}} \,. \tag{A.12}$$

Therefore in this setup the IR space below this region does not play important role for inflation because the brane velocity cannot further decrease. Accordingly, for inflation to happen, we need a large mass-squared in the moduli potential to give a high inflationary energy. This requires (A.10) because the total e-folds is given by

$$N_{\rm tot} = p \ln \left(\frac{r_i}{r_f} \right) , \tag{A.13}$$

where $r_i > r > r_f$ is the valid region for the behavior (A.5), $r_i/r_f \sim N_S^{1/2}/n^{1/2}$. To have N_e e-folds of inflation, we need $m_S \gtrsim N_e M_{\rm Pl}/\sqrt{N_S n}$. This may rely on moduli potentials. (It is easy to check that the constant vacuum energy provided by antibranes sitting in the IR end is negligible to the inflation.) The Hubble constant is time-dependent in this case, $H \approx p/t$, and the inflation is in power law, $a(t) \propto t^p$. It may be interesting to apply the rescaling to the region $r < r_f$.

B. A multi-throat slow-roll model

In this appendix, we study the case $\beta \lesssim 1$ in a repulsive B-throat. We show that it is generally a combination of the DBI and slow-roll inflation. The resulting multi-throat slow-roll model is studied. This appendix has some overlap with an independent paper recently appeared [31].

We start with the slow-roll case. The scalar velocity is determined by the non-relativistic equation of motion (2.11) by neglecting the first term,

$$\dot{r} \approx -\frac{V'}{3H} \,. \tag{B.1}$$

This procedure is valid only when the slow-roll condition is satisfied, $\beta \ll 3$. This velocity reaches the speed-limit h^2 at

$$r \approx \frac{\beta H R_B^2}{3} \,. \tag{B.2}$$

Within this slow-roll region the inflationary e-folds is given by

$$N_e = -\frac{T_3}{M_{\rm Pl}^2} \int_r^{r_m} \frac{V}{V'} dr = 3\beta^{-1} \ln \left(\frac{r_m}{r} \right) , \tag{B.3}$$

where r_m denotes the end of the flat potential and we approximate it as R_B, the extension of the throat. Taking the lower limit (B.2), we get the total number of slow-roll inflationary e-folds

$$
\begin{aligned}
(N_{\rm tot})_{sr} &\approx -3\beta^{-1}\ln(\beta H R_B) && \text{(B.4)}\\
&\approx 3\beta^{-1}\ln\left(\frac{M_{\rm Pl}}{\beta R_B T_3^{1/2}}\frac{1}{\sqrt{V_i}}\right)\\
&\sim 6\beta^{-1}\left|\ln V_i^{1/4}\right| . && \text{(B.5)}
\end{aligned}
$$

In the last step, we have neglected all terms other than V_i for simplicity.[9]

Within the relativistic region, the inflaton behaves as (2.16), but now the condition (2.17)

$$r \ll \beta H R_B^2 \tag{B.6}$$

is stronger than (2.15). This condition just matches (B.2). Taking the strongest lower bound from section 3.1, the DBI inflation then happens within

$$-\sqrt{N_B}H^{-1} < t < -\beta^{-1}H^{-1} \qquad \text{or} \qquad \frac{HR_B^2}{\sqrt{N_B}} < r < \beta H R_B^2 \,. \tag{B.7}$$

The total DBI inflationary e-folds is $\sqrt{N_B}$.

We can state the overall results by varying the parameter β. For $1 \lesssim \beta \ll \sqrt{N_B}$, we only have the DBI inflation and it lasts for $\sqrt{N_B}/\beta$ e-folds. For $0.1 \lesssim \beta \lesssim 1$, the DBI inflation still proceeds, but its end starts to deform to slow-roll, the observable universe is a mixture of both. For $0 < \beta \lesssim 0.1$, a total $6\beta^{-1}|\ln V_i^{1/4}|$ e-folds of slow-roll inflation is smoothly added to the end of a total $\sqrt{N_B}$ e-folds of DBI inflation.

Following the above discussions, we can have the following multi-throat slow-roll model. Consider n_B number of D3-branes rolling out of a B-throat, this time under a flat potential $\beta \lesssim 0.1$. (For slow-roll, the other directions have to be all lifted, so the branes do not roll towards those directions.) The inflationary energy is provided by n_A anti-D3-branes in the A-throat.

The density perturbations due to the slow-roll period can be calculated using the standard formula

$$
\begin{aligned}
\delta_H &= -\frac{T_3 V^{3/2}}{5\sqrt{3}\pi n_B^{1/2} M_{\rm Pl}^3 V'} && \text{(B.8)}\\
&\approx \frac{\sqrt{3}}{5\pi}\frac{V_i^{1/2}}{\beta\sqrt{n_B}M_{\rm Pl} r_m} e^{\beta N_e/3} \,. && \text{(B.9)}
\end{aligned}
$$

The spectral index is

$$n_s - 1 \approx -\frac{2}{3}\beta \,, \qquad \frac{dn_s}{d\ln k} \approx 0 \,. \tag{B.10}$$

[9]Even if β is not very small as long as it satisfies the slow-roll condition, for example $\beta \approx 0.3$, a long period of slow-roll inflation can be achieved because of the factor $|\ln V_i^{1/4}|$. For example, if V_i is supplied by an anti-D3-brane in an A-throat, $|\ln V_i^{1/4}| \approx |\ln h_A|$, which can be ~ 10. But such a period of inflation cannot be responsible for the CMB since the density perturbations that it generates is not scale invariant, $n_s - 1 \sim \mathcal{O}(\beta)$.

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The density perturbations and spectral indices of the slow-roll inflation and the proceeding DBI inflation should be smoothly connected to each other in the transition region. This can be seen by evaluating (B.9) at $N_e \sim (N_{\rm tot})_{sr}$ given in (B.4). In terms of the time delay (2.22), both of them have the same δr_*, while $\dot{r}_*$ transits through (B.2). As N_e increases, the density perturbation changes from $\sim e^{\beta N_e/3}$ to $\sim N_e^2$, so is growing, and then get suppressed at (5.12).

From eq. (B.9), we can see that the warp factor of the A-throat cannot be as small as the RS ratio $\sim e^{-30}$, because $V_i^{1/2} = \sqrt{2n_A} h_A^2$ while $\delta_H \approx 1.9 \times 10^{-5}$. The rescaling mechanism does not help here either (but can happen). For example suppose V_i is dominated by a kink in the bulk moduli potential and the branes gain too much kinetic energy so rescaling happens in the A-throat. This introduces a factor $\alpha^{1/2} N_A^{1/4} h_A / V_i^{1/4}$ as in (5.10). Hence $\delta_H \propto h_A V_i^{1/4}$, still too small if $h_A \sim e^{-30}$.

So let us consider situations similar to Case C in section 5. For example, using $\beta \approx 0.01$, $r_m \approx R_B$, $N_B \approx 10^4$, $N_e \approx 60$ and $n_A \approx n_B$, we get

$$\delta_H \approx 3.5 \frac{T_3^{1/4}}{M_{\rm Pl}} h_A^2 \,. \tag{B.11}$$

If $T_3/M_{\rm Pl}^4 \sim 10^{-3}$, we have $h_A \sim 5 \times 10^{-3}$.

The cosmic string tension in the A-throat is

$$G\mu_F = \sqrt{\frac{g_s}{32\pi}} \frac{T_3^{1/2} h_A^2}{M_{\rm Pl}^2} \tag{B.12}$$

$$\approx 0.008\ \delta_H^2 h_A^{-2} \sqrt{g_s} \,. \tag{B.13}$$

It can take a wide range of value. The upper bound comes from the experimental tensor modes bound on the inflationary energy $V_i T_3 = 2n_A h_A^4 T_3$ and we take it to be $V_i T_3 / M_{\rm Pl}^4 < 1.7 \times 10^{-8}$ as in (5.19). From (B.12),

$$G\mu_F < 9 \times 10^{-6} \sqrt{\frac{g_s}{n_A}} \,. \tag{B.14}$$

The lower bound comes by setting $h_A \sim 1$ in (B.13),

$$G\mu_F \gtrsim 3 \times 10^{-12} \sqrt{g_s} \,. \tag{B.15}$$

This range is within the observational ability. The strings in various throats with (5.22), left over from the Hagedorn transition from the dS epoch, have tension (5.25).

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